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LUMINESCENCE OF TWO-PHASE INHOMOGENEOUS MEDIA OF CYLINDRICAL GEOMETRY

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Abstract – An iterative rapidly convergent method for solving the radiation transfer equation is suggested. The adequate accuracy of the previously derived formula for the emissivity of a two-phase medium of cylindrical geometry is shown and the emissivity nomograms are presented. An analysis is made of the limits of applicability of a one-dimensional approach to calculate the luminescence of nonisothermal media of cylindrical geometry.

NOMENCLATURE

I, $= I(\tau, \theta, \phi)$, radiation intensity at point τ and in direction $\mathbf{l} = \mathbf{l}(\theta, \phi)$;

B, $= B_\nu(T) = \dfrac{2h\nu^3}{c^2}(e^{h\nu/kT}-1)^{-1}$, Planck radiation intensity for frequency ν and temperature T;

J, $= J(\tau) = \dfrac{1}{4\pi}\displaystyle\int_{(4\pi)} I(\tau, \theta, \phi)\,d\Omega$, averaged radiation intensity;

S, $= S(\tau)$, function of radiation sources;

k, σ, absorption and scattering coefficient, respectively;

α, $= k+\sigma$, attenuation coefficient of the medium;

λ, $= \dfrac{\sigma}{k+\sigma}$, probability of quantum survival (or the Schuster number);

T_0, temperature along the cylinder axis;

T_s, surface temperature;

$0 \leqslant r \leqslant R$, cylinder radius;

$0 \leqslant \tau = \displaystyle\int_0^r \alpha\,dr \leqslant \tau_0 = \int_0^R \alpha\,dr$, optical radial thickness of the cylinder;

$\bar{I}_n(x)$, n-order Bessel function of imaginary argument.

1. INTRODUCTION

OPTIMIZATION of modern power plants associated with a considerable increase in the temperature of the coolant, which is usually a mixture of gases and the condensed phase particles, requires as precise determination of their thermodynamic characteristics as possible. In this case the fraction of radiation in the total energy balance becomes appreciable, and it is imperative, therefore, to have exact solution of the radiative heat-transfer problems. On the other hand, it is necessary to justify the radiative transfer equation for more real physical models and determination of the limits of applicability of its solutions, and on the other, to use reliable spectroscopic characteristics of the media under study. The solution of radiative heat-transfer problems plays an important part in the study of the entry of space vehicles into the atmosphere of the Earth and other planets, in the study of interaction of high-power (laser) radiation with the material, in calculating the heating of the bases of propulsors, visibility of the flames of power plants, and of the energy balances of planetary atmospheres etc.

At present a considerable amount of attention is devoted to radiation propagating in nonplane two-phase media. The plane layer approximation for the radiative transfer problems within the optical thickness $\tau < 3$ may introduce appreciable errors of above 30–40% [1]. Radiation propagating in two-phase nonplane media is handled by the approximate methods [1–3], improved Monte-Carlo methods [4, 5], iterative techniques [6, 7], as well as by the methods of reducing the initial integro-differential equation to the integral ones with subsequent numerical integration [8–10]. It should be noted that a search for the approximate methods to calculate thermal behaviour of two-phase nonplane media is expedient both from the viewpoint of rapid radiation estimates, and from the point of view of determining the most suitable first iteration in numerical calculations of the radiative transfer equation.

In this paper the iterative method is suggested to solve the radiative transfer equation for two-phase media of cylindrical geometry which is based on approximate solution technique developed in the previous work [1]. In addition, the limits of applicability of this approximate solution have been ascertained and an analysis has been made to establish the possibility of its application to study the luminescence characteristics of two-phase nonisothermal cylindrical media by averaging the temperature and optical characteristics of the medium by a variety of techniques.

The equation of radiation transfer in a two-phase cylindrical medium is of the form [11]

$$\sin\theta\left(\cos\phi\frac{\partial I}{\partial\tau} - \frac{\sin\phi}{\tau}\frac{\partial I}{\partial\phi}\right) + I(\tau,\theta,\phi) = S(\tau), \tag{1}$$

where

$$S(\tau) = \lambda J(\tau) + S_0(\tau). \tag{2}$$

The function $S_0(\tau)$ represents distribution of the radiative sources in the medium being studied. In the case of local thermodynamic equilibrium, $S_0(\tau) = (1-\lambda)B(T)$. In equation (1) a spherical indicatrix of radiation scattering on a volume element is assumed. This assumption may be considered justified for the case of multiple scattering if the scattering indicatrix is expressed as [1, 6, 12]

$$p(\mathbf{l}, \mathbf{l}') = a + 4\pi(1-a)\delta(\mathbf{l}-\mathbf{l}'). \tag{3}$$

This expression produces a radiative transfer equation with a spherical indicatrix of scattering by performing substitution of $a\sigma$ for σ. The quantity a has the meaning of twice the semi-spherical fraction of backward scattering when radiation interacts with a volume element of the substance.

For zero boundary conditions the solution of equation (1) for a cylindrical homogeneous medium has been suggested in [1] as

$$J(\tau) = B[1 - A\bar{I}_0(k\tau)], \tag{4}$$

$$I(\tau, \theta, \phi) = B\left\{1 - \exp\left[-\frac{(\tau_0^2 - \tau^2 \sin^2\phi)^{1/2} + \tau\cos\phi}{\sin\theta}\right] - \lambda A \exp\left(-\frac{\tau\cos\phi}{\sin\theta}\right)\int_{-(\tau_0^2-\tau^2\sin^2\phi)^{1/2}}^{\tau\cos\phi} \bar{I}_0[k(x^2+\tau^2\sin^2\phi)^{1/2}]\exp\left(\frac{x}{\sin\theta}\right)\frac{dx}{\sin\theta}\right\}, \tag{5}$$

where

$$k = [3(1-\lambda)]^{1/2}; \quad A^{-1} = \bar{I}_0(k\tau_0) + \tfrac{2}{3}k\bar{I}_1(k\tau_0). \tag{6}$$

From equation (5), an analytical expression may be obtained for the emissivity of a two-phase cylindrical medium [1]

$$\varepsilon = \varepsilon(\tau_0, \theta) = 1 - \exp\left(-\frac{\tau_0}{\sin\theta}\right) - \frac{6\lambda\left(2 - \dfrac{k\tau_0\delta}{3k\tau_0+\delta^2}\right)\left[1 - \exp\left(-\dfrac{k\tau_0}{\delta}\right)\right]}{(4 + k\sin\theta)\left\{3 + 2k\left[1 - \exp\left(-\dfrac{k\tau_0}{2}\right)\right]\right\}}, \tag{7}$$

in which

$$\delta = \frac{4k\sin\theta}{4 + k\sin\theta}.$$

2. ALGORITHM STRUCTURE

For the purpose of performing numerical integration of equation (1), the iterative procedure has been used. Preliminary calculations have indicated that the best choice for the first iteration is equation (4). By introducing the variables $\gamma = \cos\theta$ and $\mu = \cos\phi$ and taking into account the axial symmetry of the medium, equation (1) may be written as

$$(1-\gamma^2)^{1/2}\left(\mu\frac{\partial I}{\partial\tau} + \frac{1-\mu^2}{\tau}\frac{\partial I}{\partial\mu}\right) + I = \frac{\lambda}{\pi}\int_{-1}^{1}\frac{d\mu'}{(1-\mu'^2)^{1/2}}\int_0^1 I(\tau, \mu', \gamma')\,d\gamma' + (1-\lambda)B(\tau). \tag{8}$$

The boundary conditions for this above equation are

$$I(\tau_0, \mu, \gamma) = 0 \quad \text{at} \quad -1 \leqslant \mu \leqslant 0. \tag{9}$$

Now, let us introduce $\gamma_k \in [0, 1]$ $(k = 1, 2, \ldots)$ which are the abscissas of Gauss' quadrature expression in γ for the integral on the RHS of equation (8). Then equation (8) may be replaced by

$$\left.\begin{aligned} &\mu\frac{\partial I_k}{\partial\tau} + \frac{1-\mu^2}{\tau}\frac{\partial I_k}{\partial\mu} + \frac{1}{(1-\gamma_k^2)^{1/2}}I_k = S_k \\ &I_k(\tau_0, \mu) = 0, \quad \text{at} \quad -1 \leqslant \mu \leqslant 0 \end{aligned}\right\}, \tag{10}$$

where $I_k \equiv I(\tau, \mu, \gamma_k)$,

$$S_k = \frac{1}{(1-\gamma_k^2)^{1/2}}\left[\frac{\lambda}{\pi}I_0(\tau) + (1-\lambda)B(\tau)\right], \quad I_0(\tau) = \int_{-1}^{1}\frac{d\mu'}{(1-\mu'^2)^{1/2}}\int_0^1 I(\tau, \mu', \gamma')\,d\gamma'. \tag{11}$$

On changing the variables

$$x = \tau\mu \quad \text{and} \quad y = \tau(1-\mu^2)^{1/2} \tag{12}$$

the domain $[0 \leqslant \tau \leqslant \tau_0,\ -1 \leqslant \mu \leqslant 1]$ will reduce to a semi-circle $[0 \leqslant y \leqslant \tau_0,\ -\tau_0 \leqslant x \leqslant \tau_0]$, with the operator $\mu(\partial/\partial\tau) + [(1-\mu^2)/\tau](\partial/\partial\mu)$ being brought to the form $\partial/\partial x$. The lines $\tau(1-\mu^2)^{1/2} = \text{const.}$, which are the characteristics of the differential operator in (10), transform into the characteristics $y = \text{const.}$ of the differential operator $\partial/\partial x$. Thus, instead of the system (10), we have

$$\left.\begin{aligned} &\frac{\partial I_k}{\partial x}+\frac{1}{(1-\gamma_k^2)^{1/2}}I_k=S_k \\ &I_k[-(\tau_0^2-y^2)^{1/2},y]=0 \ \text{at} \ 0=y\leqslant\tau_0, \quad k=1,2,\ldots,n. \end{aligned}\right\} \tag{13}$$

The grid used for the solution of the above system of equations is shown in Fig. 1. At nodal points along the characteristics y_i = const., the solution of equations (13) is of the form

$$I_{k,i,j}=I_{k,i-1,j}\cdot q_{k,i}+\exp\left[-\frac{x_i}{(1-\gamma_k^2)^{1/2}}\right]\int_{x_{i-1}}^{x_i}S_k\exp\left[\frac{x}{(1-\gamma_k^2)^{1/2}}\right]\mathrm{d}x, \tag{14}$$

where

$$I_{k,i,j}\equiv I(x_i,y_j,\gamma_k),\quad q_{k,i}=\exp\left[-\frac{\Delta x_i}{(1-\gamma_k^2)^{1/2}}\right],\quad \Delta x_i=x_i-x_{i-1}.$$

Assuming, approximately, that

$$\frac{\partial S_k}{\partial x}\cong\frac{1}{\Delta x_i}[S_k(x_i)-S_k(x_{i-1})]$$

we may evaluate the integral on the RHS of relation (14)

$$\begin{gathered} I_{k,i,j}=I_{k,i-1,j}\cdot q_{k,i}+S_{k,i-1,j}(1-\gamma_k^2)^{1/2}p_{k,i}+(1-q_{k,i}-p_{k,i})S_{k,i,j}(1-\gamma_k^2)^{1/2}, \\ p_{k,i}=\frac{1}{\Delta x_i}(1-q_{k,i})(1-\gamma_k^2)^{1/2}-q_{k,i}. \end{gathered} \tag{15}$$

Let us now represent the integral $I_0(\tau)$, equation (11), in the form of the Gaussian sum

$$I_0(\tau)=\int_0^1\mathrm{d}\gamma'\int_{-1}^1\frac{I(\tau,\mu',\gamma')}{(1-\mu'^2)^{1/2}}\mathrm{d}\mu'\cong\sum_{k=1}^n A_k\mathscr{F}(\tau,\gamma_k), \tag{16}$$

where

$$\mathscr{F}(\tau,\gamma_k)=\sum_m\int_{\mu_{m-1}}^{\mu_m}\frac{I(\tau,\mu',\gamma_k)}{(1-\mu'^2)^{1/2}}\mathrm{d}\mu'.$$

Here, the weights A_k and the nodes γ_k coincide with the positive weights and nodes in the quoted Gaussian quadrature expression in γ for the interval $[-1, 1]$.

Assuming that I is linearly dependent on μ in the interval $[\mu_{m-1}, \mu_m]$, i.e.

$$\begin{gathered} I(\tau,\mu,\gamma_k)=\frac{\mu-\mu_{m-1}}{\mu_m-\mu_{m-1}}I_m+\frac{\mu_m-\mu}{\mu_m-\mu_{m-1}}I_{m-1}, \\ I_m\equiv I(\tau,\mu_m,\gamma_k) \end{gathered} \tag{17}$$

the functions $\mathscr{F}(\tau,\gamma_k)$ in equation (16) can be given in the form

$$\mathscr{F}(\tau,\gamma_k)=\sum_m(B_mI_m+C_{m-1}I_{m-1}), \tag{18}$$

where

$$\begin{gathered} B_m=-\frac{1}{\Delta x_m}\left[\Delta y_m+x_{m-1}\left(\arcsin\frac{x_m}{\tau}-\arcsin\frac{x_{m-1}}{\tau}\right)\right], \\ C_{m-1}=\frac{1}{\Delta x_m}\left[\Delta y_m+x_m\left(\arcsin\frac{x_m}{\tau}-\arcsin\frac{x_{m-1}}{\tau}\right)\right], \\ \Delta x_m=x_m-x_{m-1},\quad \Delta y_m=y_m-y_{m-1}. \end{gathered} \tag{19}$$

This allows equation (16) to be written as

$$I_0(\tau)=\sum_k A_k\sum_m[B_mI(\tau,\mu_m,\gamma_k)+C_{m-1}I(\tau,\mu_{m-1},\gamma_k)]. \tag{20}$$

Relations (14) and (20) make it possible to describe the structure of the algorithm to solve the radiative transfer problem for a two-phase cylindrical medium under the condition of local thermodynamic equilibrium. For each point τ_i, the value of $I_0(\tau_i)=\pi J(\tau_i)$ is calculated by formula (4) with subsequent calculation of $S_k(\tau)$ by formula (11). Accounting for the boundary conditions (13), relation (15) is calculated for all the nodal points of the grid (Fig. 1). The resulting intensity values are substituted into formula (20) to determine the integral term of the transfer equation in a new approximation. The iteration is carried out until the solution of the preassigned accuracy is obtained.

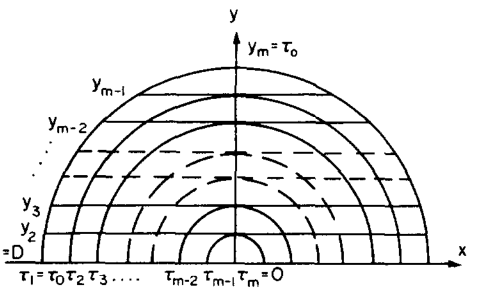

FIG. 1. Choice of nodal points for solution of the system of equations (13).

0.95. With increase in the optical radius of the cylinder, relation (7) yields substantially more accurate results. At $\lambda \leqslant 0.9$ and $\tau_0 \geqslant 0.1$, the error is estimated to be practically within 20–25%. It should be noted that relation (5) yields much greater accuracy, as is evident from Table 1. In addition, the tabulated data define the dependence of the luminescence intensity of a two-phase cylinder on the angle ϕ and confirm the correctness of its calculation performed in [1].

On the basis of the numerous calculations carried out, emissivity nomograms have been constructed which are given in Fig. 2. The curves for small optical thicknesses and large probabilities of quantum survival have been corrected by accurate calculations.

Table 1. Comparison between the approximate solution $I(\tau_0, \theta, \phi)/B$, equation (5), and exact values of $I_T(\tau_0, \theta, \phi)/B$ ($\lambda = 0.95$)

θ	τ_0	0.1		0.5		1.0		5.0		10.0	
	ϕ	I/B	I_T/B	I/B	I_T/B	I/B	I_T/B	I/B	I_T/B	I/B	I_T/B
	0°	0.0105	0.0107	0.052	0.054	0.101	0.106	0.341	0.339	0.404	0.401
	15°	0.0102	0.0104	0.050	0.053	0.099	0.103	0.336	0.332	0.399	0.395
	30°	0.0092	0.0094	0.047	0.048	0.093	0.096	0.319	0.318	0.382	0.379
70°	45°	0.0077	0.0077	0.040	0.041	0.082	0.083	0.292	0.291	0.353	0.349
	60°	0.0055	0.0055	0.031	0.031	0.065	0.064	0.254	0.250	0.313	0.306
	75°	0.0031	0.0031	0.0187	0.0182	0.043	0.041	0.205	0.194	0.265	0.251
	85°	0.0010	0.0010	0.0065	0.0062	0.016	0.015	0.118	0.106	0.193	0.174
	0°	0.0179	0.0182	0.066	0.071	0.110	0.116	0.285	0.283	0.333	0.331
	15°	0.0174	0.0177	0.066	0.070	0.109	0.114	0.282	0.280	0.330	0.325
	30°	0.0159	0.0161	0.063	0.066	0.105	0.109	0.272	0.270	0.320	0.314
30°	45°	0.0133	0.0134	0.057	0.058	0.098	0.099	0.256	0.252	0.302	0.296
	60°	0.0098	0.0098	0.047	0.047	0.086	0.085	0.235	0.229	0.279	0.270
	75°	0.0056	0.0056	0.031	0.030	0.064	0.061	0.212	0.200	0.252	0.236
	85°	0.0018	0.0018	0.011	0.011	0.027	0.026	0.160	0.144	0.221	0.198

The above algorithm was used to compose the program coded in FORTRAN-IV. Numerical calculations were done on the "EC-1030" and "EC-1022" computers. The results have demonstrated the effectiveness of the method suggested. The accuracy of 0.1% for $\tau_0 \sim 0.01-0.1$ is attained after 2–3 iterations and for $\tau_0 \sim 1$, after $\lesssim 10$ iterations, and then as τ_0 rises, the number of iterations increases and at $\tau_0 \sim 15$ it amounts to about 100.

The program provides for the division of the interval of integration over $\phi[0, \pi/2]$ into 13 parts, and over $\theta[0, \pi/2]$, into 3 parts.

In choosing the number of the division points, the accuracy control was made as this number increased (up to 22 points for ϕ and up to 8 points for θ).* Integration over ϕ for $\tau_0 = 10$ introduces an error of about 2% at $\lambda = 0.3$ and of about 12% at $\lambda = 0.999$, while integration over θ results in the error of about 3% at $\tau_0 = 0.01$ which decreases greatly with increasing τ_0.

3. ACCURACY OF THE APPROXIMATE RELATIONS (5) AND (7)

Comparison between the accurate calculations and predictions by relation (7) indicates that the results show a satisfactory agreement for $\lambda \leqslant 0.90$–

*The weights and nodes of Gauss' quadrature formula have been taken according to [13].

The calculated luminescence indicatrix for a two-phase cylindrical medium, presented in Fig. 3, exhibits a considerable variation when passing from small to large optical thicknesses, which is the fact originally established in [1]. Thus, the degree of the luminescence indicatrix anisotropy

$$r = \frac{I(\tau_0, \theta)|_{\theta=\pi/2}}{I(\tau_0, \theta)|_{\theta=0}}$$

amounts to about 10 at $\tau_0 = 0.05$ and $\lambda = 0.999$, to 0.8 at $\tau_0 = 1.0$ and is only 0.56 at $\tau_0 = 10$.

4. LUMINESCENCE OF A NONISOTHERMAL TWO-PHASE CYLINDER AND POTENTIALITY OF A ONE-DIMENSIONAL APPROXIMATION TECHNIQUE

In the majority of cases the two-phase media are nonisothermal, i.e. $B(T) = B[T(r)]$. To study the luminescence characteristics of such media having cylindrical configuration, we have assumed the following dependence of temperature T and the absorption coefficient κ on the optical thickness of the cylinder

$$T(\tau) = T_0 \exp(-\alpha\tau^2), \quad \kappa(\tau) = \kappa_0 \exp(-\alpha\tau^2), \qquad (21)$$

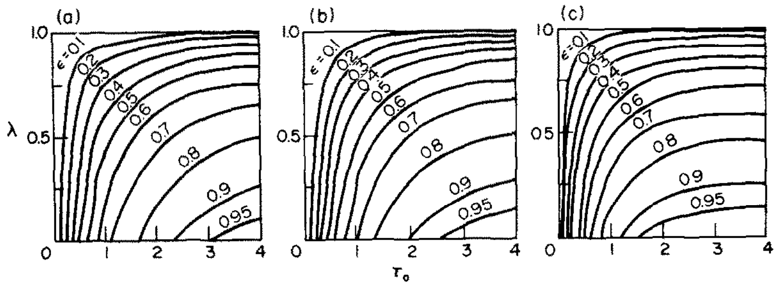

FIG. 2. Emissivity nomograms for a two-phase cylindrical medium: (a) $\theta = 90°$; (b) $\theta = 60°$; (c) $\theta = 30°$.

where $T_s = T(\tau_0) = 300$ K, and T_0 ranges from 500 to 2000 K. In addition to numerical calculations with regard for relations (21), the possibility has been studied of using relation (7) when different techniques of averaging the temperature and the absorption coefficient are employed, i.e. the possibility of one-dimensional modeling of a nonisothermal two-phase cylinder. The averaging has been performed as follows

(a) $$\bar{T} = \frac{1}{R}\int_0^R T(r)\,\mathrm{d}r,$$

(b) $$B(\bar{T}) = \frac{1}{R}\int_0^R B[T(r)]\,\mathrm{d}r,$$

(c) $$\bar{T} = \frac{\int_0^R \kappa(r)T(r)\,\mathrm{d}r}{\int_0^R \kappa(r)\,\mathrm{d}r},$$

(d) $$B(\bar{T}) = \frac{\int_0^R \kappa(r)B[T(r)]\,\mathrm{d}r}{\int_0^R \kappa(r)\,\mathrm{d}r},$$

(e) $$\bar{\kappa} = \frac{\int_0^R \kappa(r)T(r)\,\mathrm{d}r}{\int_0^R T(r)\,\mathrm{d}r},$$

(f) $$\bar{\kappa} = \frac{\int_0^R \kappa(r)B[T(r)]\,\mathrm{d}r}{\int_0^R B[T(r)]\,\mathrm{d}r}.$$

The predicted data for the intensity of radiation emitting from a two-phase cylinder, which have been obtained by performing numerical integration of the transfer equation and using a one-dimensional approximation, are presented in Figs. 4 and 5. It may there be seen that the best coincidence is observed at small optical thicknesses and, moreover, the uncertainty introduced by relation (7) diminishes considerably with a decrease in the probability of quantum survival. Of the quoted averaging techniques (a–d; e, f), the technique (a, e) is preferable. With increase in the optical thickness, the one-dimensional representation results in larger errors since radiation from a hot central region scarcely reaches the boundary surface. Figure 4 shows also

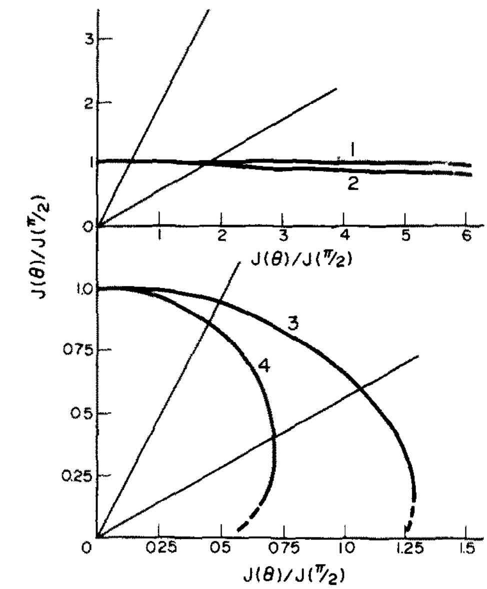

FIG. 3. Luminescence indicatrix of a two-phase cylinder at $\lambda = 0.999$. (1) $\tau_0 = 0.01$; (2) 0.05; (3) 1.0; (4) 10.0.

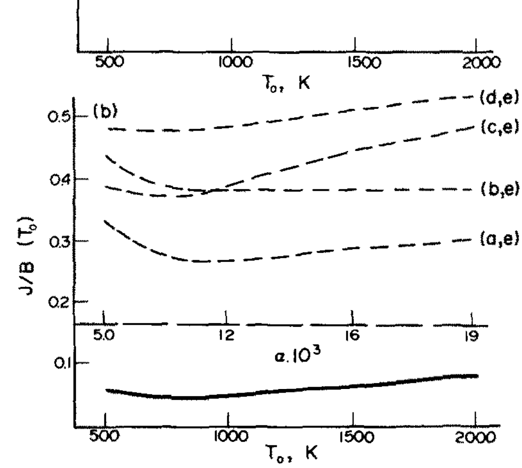

FIG. 4. Comparison of the accurate predictions and different averaging techniques for determining radiation of two-phase cylinders ($\lambda = 0.3$). (a) $\tau_0 = 0.01$; (b) 10.0.

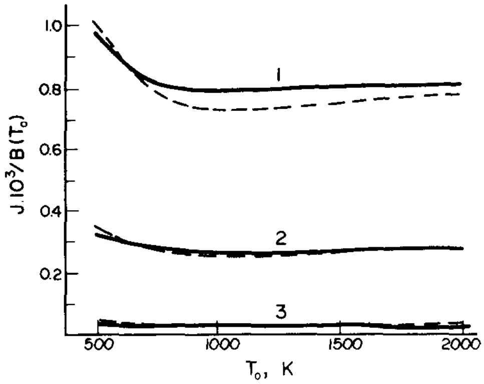

FIG. 5. Comparison of the accurate predictions and the averaging technique (a, e) at different probabilities of photon survival $(\tau_0 = 0.01)$. (1) $\lambda = 0.7$; (2) 0.9; (3) 0.99.

the effect of the parameter α which characterizes the temperature field gradient in the medium under study.

5. CONCLUSION

The suggested iterative method is very effective in the case of a two-phase medium of cylindrical geometry whose luminescence characteristics are to be calculated. Accurate calculations of the emissivity have allowed determination of the limits of applicability for the approximate relation (7) which was derived in [1]. The emissivity nomograms for a two-phase cylinder are very practical when the spectral relationships of the luminescence characteristics are required. In the case of nonisothermal media, the approximate relations used for homogeneous media with averaging of the physical quantities may introduce appreciable errors. Here it is necessary to resort to the accurate numerical methods, one of these being described in the present paper.

REFERENCES

1. K. S. Adzerikho and V. P. Nekrasov, Luminescence characteristics of cylindrical and spherical light-scattering media, *Int. J. Heat Mass Transfer* **18**, 1131–1138 (1975).
2. J. T. Jefferies, On the diffusion of radiation from a point or a line source in an infinite medium, *Optica Acta* **2**(3), 109–114 (1955).
3. R. G. Giovanelli and J. T. Jefferies, Radiative transfer with distributed sources, *Proc. Phys. Soc.* **B69**, part II, **443B**, 1077–1084 (1956).
4. G. J. Marchuk, G. A. Mikhailow, M. A. Nazariliev and R. A. Darbimyan, *Solution of Direct and Some Inverse Problems on Atmospheric Optics by the Monte Carlo Method.* Nauka, Novosibirsk (1968).
5. D. G. Collins, W. G. Blattner, M. B. Wells and H. G. Horak, Backward Monte Carlo calculation of the polarization characteristics of the radiation emerging from spherical-shell atmosphere, *Appl. Optics* **11**, 2684–2696 (1972).
6. L. P. Bass, T. A. Germogenova *et al.*, One-speed program "Raduga-1", Preprint No. 11, Inst. Prikl. Mat. AN SSSR, Moscow (1973).
7. A. V. Prokopchenko, On higher-order iteration processes, *Zh. Vych. Mat. i Mat. Fiz.* **14**, 230–233 (1974).
8. J. N. Minin and V. V. Sobolev, Light scattering in spherical atmosphere, Parts II and III, *Cosmic Studies* **1**(2), 227–234 (1963); **2**(4), 610–618 (1964).
9. N. B. Engibaryan, Radiative transfer in a spherical medium, *Uch. Zap. Erevan. Univ., Ser. Estestv. Nauk* **1**, 26–37 (1972).
10. S. J. Wilson and K. K. Sen, Probabilistic model for the resolvent kernel in diffusion problems in spherical-shell media, *J. Quantve Spectros & Radiat. Transf.* **13**, 255–266 (1973).
11. A. Uesugi and J. Tsujita, Diffuse reflection of a searchlight beam by slab, cylindrical and spherical media, *Publ. Astron. Japan* **21**(4), 370–383 (1969).
12. K. S. Adzerikho and V. P. Nekrasov, Calculation of luminescence characteristic of light-scattering media, Parts I, II, *Inzh.-Fiz. Zh.* **22**(1), 168–170 (1972).
13. A. N. Lowan, N. Davids and A. Levenson, Table of the zeros of the Legendre polynomials of order 1–16 and the weight coefficients for Gauss' mechanical quadrature formula, *Bull. Am. Math. Soc.* **48**, 739–743 (1942).

LUMINESCENCE D'UN MILIEU BIPHASIQUE NON HOMOGENE ET DE GEOMETRIE CYLINDRIQUE

Résumé—On propose une méthode itérative rapidement convergente pour résoudre l'équation de transfert radiatif. On montre quelle est la précision des formules antérieures pour l'émissivité d'un milieu diphasique à géométrie cylindrique et on présente les nomogrammes d'émissivité. On analyse les limites d'application d'une approche monodimensionnelle pour calculer la luminescence d'un milieu non isotherme de géométrie cylindrique.

LUMINESZENZ VON INHOMOGENEN ZWEI-PHASEN-MEDIEN VON ZYLINDRISCHER GEOMETRIE

Zusammenfassung—Es wird eine iterative, schnell konvergierende Methode für die Lösung der Strahlungsübergangsgleichung vorgeschlagen. Die gute Genauigkeit der an anderer Stelle abgeleiteten Gleichung für das Emissionsvermögen eines Zwei-Phasen-Mediums mit zylindrischer Geometrie wird gezeigt und Nomogramme des Emissionsvermögens werden angegeben. Die Anwendungsgrenzen werden untersucht für ein eindimensionales Verfahren zur Berechnung der Lumineszenz von nicht-isothermen Medien mit zylindrischer Geometrie.

СВЕЧЕНИЕ ДВУХФАЗНЫХ НЕОДНОРОДНЫХ СРЕД ЦИЛИНДРИЧЕСКОЙ КОНФИГУРАЦИИ

Аннотация — Предложен итерационный быстро сходящийся метод решения уравнения переноса излучения. Показана удовлетворительная точность ранее полученной формулы для излучательной способности двухфазной среды цилиндрической конфигурации и приведены номограммы для ее расчета. Исследованы границы применимости одномерного расчета свечения неизотермических сред цилиндрической конфигурации.